Vagueness: other accounts

Marco Degano

Philosophical Logic 2025
5 November 2025

Readings

Suggested:

- Cobreros, P. & Tranchini, L. (2019). *Supervaluationism, Subvaluationism and the Sorites Paradox*. In Sergi Oms & Elia Zardini (eds.), *The Sorites Paradox*. New York, NY: Cambridge University Press. pp. 38–62.
- Cobreros, P., Egré, P., Ripley, D., van Rooij, R. (2015). *Vagueness, Truth and Permissive Consequence*. In: Achourioti, T., Galinon, H., Martínez Fernández, J., Fujimoto, K. (eds) Unifying the Philosophy of Truth. Logic, Epistemology, and the Unity of Science, vol 36. Springer, Dordrecht.

Further reading:

- Williamson, T. (2002). *Vagueness*. Routledge.

Outline

Supertrue vs Subtrue

- **Supervaluationism** takes a sentence to be true just in case it is true in **all** of its possible precisifications.
- **Subvaluationism** takes a sentence to be true just in case it is true in **some** of its possible precisifications.
- Supervaluationism: vagueness as *underdetermination*. Borderline cases are neither supertrue nor superfalse.
- Subvaluationism: vagueness as *overdetermination*. Borderline cases are both subtrue and subfalse.

Subtrue and Subfalse

Definition (Subtruth & Subfalsity)

Let $V \neq \varnothing$ be a set of classical valuations. Using the relation $V, v \models \varphi$ from last lecture, we define for any formula φ:

(Subtruth) $V \models^{\exists 1} \varphi \iff \exists v \in V : V, v \models \varphi$

(Subfalsity) $V \models^{\exists 0} \varphi \iff \exists v \in V : V, v \not\models \varphi$

Global and Local consequence

Definition (Global subvaluationist consequence)

$\Gamma \models_g^{\exists} \varphi$ iff for all non-empty V, if $V \models^{\exists 1} \gamma$ for all $\gamma \in \Gamma$, then $V \models^{\exists 1} \varphi$.

Definition (Local subvaluationist consequence)

$\Gamma \models_l^{\exists} \varphi$ iff for all non-empty V, if for all $\gamma \in \Gamma$ there exists $v \in V$ with $V, v \models \gamma$, then there exists $v' \in V$ with $V, v' \models \varphi$.

Yet another logical consequence

Definition (Another consequence)

$\Gamma \models^{\exists}_{\forall} \varphi$ iff for all non-empty V, if there exists $v \in V$ with $V, v \models \gamma$ for all $\gamma \in \Gamma$, then there exists $v' \in V$ with $V, v' \models \varphi$.

$$\Gamma \models^{\exists}_{\forall} \varphi \iff \Gamma \models_{CL} \varphi$$

The latter holds over the base language. For the $(\Rightarrow)$ direction take a singleton $V = \{v\}$.

Global consequence

- Global subvaluationism and classical consequence do not coincide:

$$\Gamma \models_g^{\exists} \varphi \Rightarrow \Gamma \models_{CL} \varphi$$

$$\Gamma \models_g^{\exists} \varphi \not\Leftarrow \Gamma \models_{CL} \varphi$$

- Global subvaluationism is paraconsistent:

$$\{p, \neg p\} \not\models q$$

- Note: consequence is different from LP: $p \wedge \neg p \models_g^{\exists} q$ and $\{p, q\} \not\models_g^{\exists} p \wedge q$

Sets of Conclusions

- So far we worked with single-conclusion consequence $\Gamma \models \varphi$
- **What about a set of conclusions $\Gamma \models \Phi$?**

$$\Gamma \models \Phi \quad \Longleftrightarrow \quad \neg\exists\, v\, \big(v \models \Gamma \text{ and } v \models \neg\Phi\big), \quad \text{where } \neg\Phi := \{\neg\varphi : \varphi \in \Phi\}$$

Intuition: when all premises hold, at least one member of Φ must hold (no joint countermodel).

- Multiple-conclusion arguments may not mirror ordinary inference (cf. Steinberger 2011)
- But they yield a useful connection: global sup/sub-valuationist consequence are the dual of each other:

$$\Gamma \models_g \Phi \iff \neg\Phi \models_g^{\exists} \neg\Gamma$$

Bivalence fails in sup.s: $\not\models_g \{\varphi, \neg\varphi\}$
Split-consistency fails in sub.s: $\{\varphi, \neg\varphi\} \not\models_g^{\exists}$

The Sorites

V	$\varphi(1)$	$\varphi(2)$	$\varphi(3)$	$\varphi(4)$
v_1	1	0	0	0
v_2	1	1	0	0
v_3	1	1	1	0

Conditionals $C_k : \varphi(k) \to \varphi(k+1)$*:* each C_k is true at all $v \neq v_k$ and false at v_k.

$$\forall k : V \models^{\exists 1} C_k \quad \text{but} \quad V \not\models^{\exists 1} \bigwedge_k C_k$$

- Hence every step holds somewhere (it is subtrue), but not all together at one v.
- But the *argument is invalid* since **Modus Ponens can fail** for a conditional that is both subtrue and subfalse

MP is not valid under SbV

Let $V = \{v_1, v_2\}$ with $v_1(p) = 1,\ v_1(q) = 0$ and $v_2(p) = 0,\ v_2(q) = 0$.

$$\underbrace{V \models^{\exists 1} p}_{v_1} \quad \underbrace{V \models^{\exists 1} (p \to q)}_{v_2} \quad \underbrace{V \not\models^{\exists 1} q}_{\text{no } v}$$

So $p,\ (p \to q)\ \not\models^{\exists}_{g} q$.

Each conditional C_k is subtrue (witness $v \neq v_k$) and $\varphi(1)$ is subtrue, but $\varphi(N)$ need not be subtrue. The chain of MP steps can break at the world where some C_k is false.

Universal form of the Sorites

- Consider a single line form of the argument: $\big(p_1 \wedge (p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge \ldots\big) \Rightarrow p_n$
- Then the argument is valid for subvaluationism.
- But there is no *single* v which makes all the steps true and hence the universal $\forall n\,(p_{n-1} \rightarrow p_n)$ is **not even subtrue.**
- Thus the Sorites is here blocked, even though the argument is valid.
- Subvaluationism is thus committed to different answers, depending on the form of the paradox.
- To be fair, also supervaluationism displays this asymmetry for sets of conclusions:

Supervaluationism ($\models_g$)

$$p \vee q \;\not\models_g\; \{p, q\}$$
$$\{p, q\} \;\models_g\; \{p, q\}$$

Subvaluationism ($\models_g^{\exists}$)

$$\{p, q\} \;\not\models_g^{\exists}\; p \wedge q$$
$$\{p, q\} \;\models_g^{\exists}\; \{p, q\}$$

Outline

1. Subvaluationism

2. Epistemicism

3. Logic of ST

4. Contextualism

The epistemic solution

Timothy Williamson

- **Sharp but unknowable.** Vague expressions have *sharp* extension/cutoffs. Our ignorance makes them appear vague.
- There is a precise number distinguishing *bald* from *not bald*, but **we do not know** it.
- Vagueness = **epistemic limitation**, not semantic indeterminacy.

Inexact knowledge & margins of error

- (Williamson, *Vagueness* 1994) Knowledge is **safe**: it must be stable under small changes.
- **Margin-for-error schema** (for a vague predicate P and metric d):
$$Know_{\alpha}(P(x)) \;\Rightarrow\; \forall y\big(d(x,y) \leq \alpha \;\Rightarrow\; P(y)\big)$$
- Intuition: if you *know* that x is P, then anything α-close to x must also be P. Near the sharp cutoff, this *forces* ignorance: neither *know P* nor *know not P* can hold.

Fixed margin models

A fixed-margin (Kripke) model $M = \langle W, R, V \rangle$ with metric d and error parameter $\alpha > 0$:

$$R(x, y) \text{ iff } d(x, y) \leq \alpha$$

Reading: $R(x, y)$ = "y is within the α-margin of x"

Assume for all $x, y, z \in W$:

$$d(x, y) = 0 \Leftrightarrow x = y, \qquad d(x, y) = d(y, x), \qquad d(x, z) \leq d(x, y) + d(y, z)$$

Consequences for R:

- **Reflexive** ($d(x, x) = 0 \leq \alpha$)
- **Symmetric**
- **Not necessarily transitive** ($d(x, z)$ may be 2α)

[Égré and Bonnay 2010; Williamson 2002]

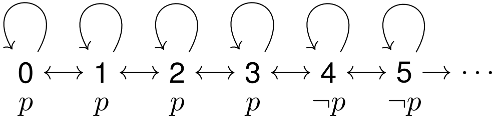
Knowledge operator

$$M, x \models \Box\varphi \quad \text{iff} \quad \forall y \, (R(x, y) \Rightarrow M, y \models \varphi)$$

- $\Box\varphi$ at x: "I **know** φ throughout the α-neighborhood of x".
- We work with a simple line model: $W = \mathbb{N}$, $d(m, n) = |m - n|$, $\alpha = 1$.

[Égré and Bonnay 2010]

Example (line model, $\alpha = 1$)



Take p as a vague predicate (e.g., “not a heap”). So small n satisfy p, large n satisfy $\neg p$.

- At 2: $\Box p$ holds (both $1, 2, 3$ satisfy p). At 5: $\Box\neg p$ holds.
- At the boundary $3, 4$: $\neg\Box p \wedge \neg\Box\neg p$ (ignorance).

Unknown sharp cutoff

- There is a **sharp cutoff** n^*. For $n < n^*$: p (not a heap). For $n \geq n^*$: $\neg p$ (heap).
- **Margin-for-error** forbids knowledge at the α-neighbors of n^*.
- At a clear case like 0, we naturally want very strong epistemic security about p: ideally $\Box p, \Box^2 p, \Box^3 p, \ldots$ all hold.
- Fixed-margin models block this ideal: they validate $\Box p$ at clear cases but *do not* generally validate all iterations $\Box^n p$. Epistemicist reply: as we iterate the knowledge operator, knowledge **"erodes"**.

Knowledge axioms

(1) $\Box\varphi \to \varphi$

Factivity: if I know φ, then φ is true.

(2) $\Box\varphi \to \Box\Box\varphi$

Positive introspection: if I know φ, then I know that I know φ.

(3) $\neg\Box\varphi \to \Box\neg\Box\varphi$

Negative introspection: if I don't know φ, then I know that I don't know φ.

- In the fixed-margin semantics, (1) is valid: knowledge is factive.
- If we also require (2) or (3) to be valid at all worlds, then the accessibility relation R must be **transitive**.
- But non-transitivity of R is exactly what allows the model to block the Sorites.
  So, on the epistemicist picture, (2) and (3) are *not* generally valid for inexact knowledge.

Assessing the epistemic response

- **Counterintuitive sharpness:** It posits **precise** cutoffs for *tall*, *heap*, etc.
- **Ignorance challenge:** If there *is* a cutoff, why can't competent speakers know it?
- **Higher-order ignorance:** Do we *know* the margin α? If not, do margins themselves admit margins? (Iterated ignorance.)

Outline

Strict vs. tolerant truth

Fix the Strong Kleene truth-functions on $\{0, i, 1\}$.

- **Strict truth** (s): $v(\varphi) = 1$.
- **Tolerant truth** (t): $v(\varphi) \neq 0$ (i.e. 1 or i).
- Duality: φ is t-true iff $\neg\varphi$ is not s-true (and vice versa).
- "strict" tracks full truth
- "tolerant" tracks non-falsity (or permissive assertability).

[Abdallah 1995; Bennett 1998; Cobreros et al. 2015]

Mixed consequence (four variants)

For $n, m \in \{\mathsf{s}, \mathsf{t}\}$:
$\Gamma \models_{nm} \varphi$ iff there is no Strong Kleene valuation v with $[\forall \gamma \in \Gamma,\ v \models_n \gamma] \ \&\ v \not\models_m \varphi$.

- $\models_{\mathsf{ss}}$ = K3 (preserves 1).
- $\models_{\mathsf{tt}}$ = LP (preserves $\neq 0$).
- $\models_{\mathsf{ts}}$ is *empty* (premises weaker than conclusions).
- $\models_{\mathsf{st}}$ = **ST**: from strict premises to tolerant conclusions.

$$\Gamma \models_{ST} \varphi \text{ iff } [\forall \gamma \in \Gamma,\ v(\gamma) = 1] \ \Rightarrow\ v(\varphi) \neq 0$$

Conditionals and why ST keeps Modus Ponens

$$v(A \to B) = \max(1 - v(A),\ v(B))$$

$$v(A \to B) \neq 0 \quad \Leftrightarrow \quad [v(A) \neq 1] \text{ or } [v(B) \neq 0].$$

So the object-language $\to$ mirrors the meta-pattern s$\Rightarrow$t:

if A is strictly true, then B is tolerantly true.

- ST validates **Modus Ponens** and the **Deduction Theorem.**

Classicality of ST (base language)

Theorem (ST = Classical Consequence)

For any Γ, φ in the base language,

$$\Gamma \models_{ST} \varphi \text{ iff } \Gamma \models_{\mathrm{CL}} \varphi$$

Prove the contrapositive:

- ($\Leftarrow$) Any classical countermodel is an ST countermodel.
- ($\Rightarrow$) Any ST countermodel can be *refined* by replacing each $\frac{1}{2}$ with 0 or 1 so as to yield a classical countermodel.

(Write out the proof in full: you may use a refinement argument from SK to classical valuations, which you need to prove by induction on formulas.)

Adding vagueness: ST with indifference (STVP)

- We extend the language with a crisp similarity predicate I_P for each vague P. With the *closeness proviso* on all valuations v:

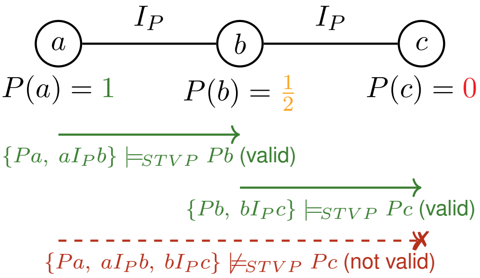
$$v \models_{\mathsf{s}} aI_Pb \quad \text{iff} \quad v \models_{\mathsf{t}} aI_Pb \quad \text{iff} \quad |v(Pa) - v(Pb)| < 1$$

- Hence I_P is reflexive symmetric, but *not necessarily transitive*.
- **Tolerance (valid in STVP):**

$$Pa,\, aI_Pb \models_{STVP} Pb \qquad\qquad \models_{STVP} \forall xy\,(Px \wedge xI_Py \rightarrow Py)$$

Intuition: if Pa holds strictly and b is P-indistinguishable from a, then Pb holds at least tolerantly.

STVP: non-transitive consequence (Sorites blocked)



- STVP validates *local* Tolerance but consequence is *non-transitive*, so the Sorites chain fails.

ST and the Sorites

- Keeps the intuitive *Tolerance* principle in conditional form.
- Blocks the paradox via *non-transitive* consequence once I_P is present.
- Preserves classical behavior (Modus Ponens, Deduction Theorem, . . .) on the base language.

Outline

Contextualism

- Vague predicates are **context-sensitive** (comparison class / standard shifts).
- The conditional premises in Sorites are *intuitively compelling*. Contextualism explains this pull via **context shifts**.

Indistinguishability Relation I

We can state the Sorites premise as:

If x is P and x is indistinguishable from y, then y is P.

$$(P(x) \ \wedge \ xI_P y) \ \rightarrow \ P(y)$$

What properties should I_P have? In particular, can I_P be *transitive*?

From "significant difference" $\succ_P$ to indistinguishability I_P

Define "significantly P-er than" by $x \succ_P y$. Then set:

$$xI_Py \;\; := \;\; \neg(x \succ_P y) \;\wedge\; \neg(y \succ_P x)$$

- If $\succ_P$ is a *strict weak order* (irreflexive, transitive, almost-connected)[1], then I_P is an **equivalence relation** (check this as an exercise). Hence transitive.
- To avoid transitivity, we use a more realistic **"just noticeable difference"** ordering.

[1] **Almost connectedness**: $\forall x \, \forall y \, \forall z \, \big(R(x,y) \rightarrow (R(x,z) \;\vee\; R(z,y))\big)$.

Semi orders

We define $\succ_P$ as a **semi-order**:

Irreflexive: $\forall x : \neg x \succ x$
Interval-order: $\forall x, y, v, w : (x \succ y \wedge v \succ w) \rightarrow (x \succ w \vee v \succ y)$
Semi-transitive $\forall x, y, z, v : (x \succ y \wedge y \succ z) \rightarrow (x \succ v \vee v \succ z)$

- Irreflexive: nothing is *noticeably* more P than itself.
- Interval-order: if we can tell x is better than y and v is better than w, then at least one “better” element also beats the other pair’s “worse” element (there are no two completely independent just-noticeable differences)
- Semi-transitive: if x is better than y and y is better than z, then any fourth option v must line up with one of the extremes, x or z

Hence I_P is reflexive and symmetric, but need not be transitive (check this as an exercise).

Context-dependent I

Contextualist move: I_P is **context-dependent** and the context *changes* along a Sorites sequence.

Similarity relativized to a comparison class $c \subseteq D$:

$$xI_P^c y \quad \text{iff} \quad \forall z \in c:\ xI_P z \leftrightarrow yI_P z$$

"x and y are not (even indirectly) distinguishable relative to c."

Local validity vs. global invalidity

Conditionals are safe *in isolation* at their own context c:

$$\big(P(x,c)\ \wedge\ xI^c_P y\big)\ \rightarrow\ P(y,c)$$

where here $c = \{x, y\}$.

But we cannot conjoin premises across *different* contexts:

(1) $P(x,c)$ with $c = \{x, y, z\}$
(2) $\big(P(x,c) \wedge xI^c_P y\big) \rightarrow P(y,c)$ with $c = \{x, y\}$
(3) $\big(P(y,c) \wedge yI^c_P z\big) \rightarrow P(z,c)$ with $c = \{y, z\}$
(4) $P(z,c)$ with $c = \{x, y, z\}$

From (1)-(3) *you cannot derive* (4): we would need (2) and (3) on $c = \{x, y, z\}$

Contextualism

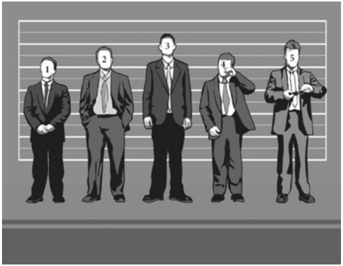
Experimental angle:

- Forced-march tasks (stepwise “still P?”) often show tolerance
- Presentation/manipulation of *comparison class* and *stimulus variation* shift judgments.

Philosophical concerns:

- Equivocation worry: Are we merely “changing the subject”?
- Fixing c: Even if we stipulate a fixed c, the paradox can still feel compelling: what explains that pull?
- Higher-order vagueness: Standards themselves seem vague. Can contextualism iterate the story?

Vagueness and Experiments



- Alxatib & Pelletier (2011): 5 men of increasing height.
- Participants judged, for each man:
  - "He is tall"
  - "He is not tall"
  - "He is tall and not tall"
  - "He is neither tall nor not tall"
- Results for man #2:
  - "He is **tall and not tall**": True 44.7%, False 40.8%, Can't tell 14.5%
  - "He is **neither tall nor not tall**": True 53.9%, False 42.1%, Can't tell 4.0%.